Interaction of Oblique Shock and Detonation Waves

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The interaction of an oblique shock wave and an oblique detonation wave which deflect the flow in the same direction is analyzed. The detonation wave is assumed to be an exothermic gasdynamic discontinuity. A criterion is developed and used to determine whether or not a theoretical solution of the problem describes a physically realizable interaction configuration. It is found that the reflected wave is a rarefaction wave; however, if the heat release parameter of the detonation wave is sufficiently small, then the reflected wave can be a shock wave. Domains of existence of such resulting wave interaction configurations are established for different values of the oncoming Mach number, $6 \leq M \leq 8$, the heat release parameter, $3 \leq Q \leq 8$, and the specific heat ratios for the combustion products behind the detonation wave, $1.30 \leq \gamma \leq 1.33$. It was also found that double-discontinuity configurations, representing the refraction of a detonation wave at a combustible/noncombustible interface (a limiting case of the considered interaction problem) can exist for certain values of the flow parameters involved and for different specific heat ratios of the gases in front of and behind the detonation wave. The magnitudes of the heat release parameter and specific heat ratio of the combustion products affect significantly the interaction pattern of shock and detonation waves. It is, therefore, concluded that analysis of this interaction problem should take into account the detailed thermodynamics of the combustible mixture of gases.

Nomenclature

a	= sound speed
c_p	= specific heat at constant pressure
c_v	= specific heat at constant volume
M	= Mach number
M_n	= normal Mach number
P_{ij}	= pressure ratio, $= p_i/p_j$
q_{ij}	= heat release in the wave separating flow regions i and j
Q_{ij}	= heat release parameter, $= q_{ij}/c_{pj}T_j$
R	= gas constant
T	= absolute temperature
u	= component of flow velocity normal to discontinuity surface
v	= component of flow velocity tangential to discontinuity surface
V	= flow velocity
ρ	= density
γ	= specific heat ratio
δ	= flow deflection angle through a wave

Subscripts

i	= downstream of a wave
j	= upstream of a wave
CJ	= Chapman-Jouguet condition
ℓ	= lower limit
u	= upper limit

Superscripts

0	= stagnation values of the corresponding flow variables

Introduction

THE use of oblique detonation waves in ramjets that operate at hypersonic speeds have recently received considerable attention.[1,2] A flow device utilizing this mode of combustion, as opposed to the supersonic diffusive burning mode, offers several advantages (see, for example, Ref. 3). Proposed diffuser-combustor model flow configurations involving plane oblique detonation waves[1,2,4] by necessity entail the interaction of such combustion waves with the oblique shock waves formed on the forebody at high supersonic flight speeds. In view of such possible applications, the shock wave and detonation wave interaction problem depicted in Fig. 1 is investigated in the present study. Fuel is injected along the wall AB of the inlet into the supersonic stream and is assumed fully mixed with the air at station B. A detonation wave across BO, stabilized by means of a second wedge, interacts with the oblique shock wave AO formed by the forebody. Of primary interest are the nature and magnitude of the resulting possible wave configurations, as well as their domains of existence as functions of the oncoming flow Mach number, the strengths of the shock and detonation waves, the amount of heat released by the burning reaction per unit mass of gas, and the specific heat ratios of the combustion product behind the detonation wave.

It is easy to visualize that in order to equilibrate the pressures in the airstreams on both sides of the intersection point O, the transmitted discontinuity must be a shock wave OE; a contact discontinuity OC must then separate the airstreams of different entropies passing through the two different sets of discontinuities. However, such a "triple-discontinuity" configuration where only a reflected Mach wave exists represents a special limiting case of the more general resulting flow pattern when a reflected shock or rarefaction wave is present between the detonation wave OB and the contact surface OC. Figures 1b and 1c represent the resulting discontinuity patterns when the reflected wave is a rarefaction wave and a shock wave, respectively. It is easy to show that there can be no other discontinuities in the interaction flow pattern, i.e., no discontinuity (other than the reflected rarefaction or shock wave) can exist between the detonation wave and the contact surface; also, no other discontinuity can exist between the transmitted shock wave OE and the contact surface OC (see Ref. 5, Chap. 11).

The analysis assumes that the detonation wave satisfies the Chapman-Jouguet jump conditions. According to this model the detonation wave consists of a shock wave in which chemical reactions occur instantaneously, i.e., the detonation wave is considered as an exothermic gasdynamic discontinuity. Although for most fuels utilized in ramjets the temperature behind the detonation wave would be too high to consider the flowing medium as a perfect gas, we have neglected real-gas effects for the sake of simplicity. The effects of chemical reactions, as well as of the various fuel/air mixture ratios are accounted for by different constant specific heat ratios before and after the detonation wave and by different values of the heat release parameter.

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The shock and detonation wave interaction was also investigated by Rues[6] for the particular case when the resulting wave configuration contains only a reflected Mach wave; no allowance was made for the change of specific heat ratios across the detonation wave. A qualitative analysis of a limiting case of the interaction problem, i.e., the refraction of a detonation wave at a combustible/noncombustible gas interface, is given in Ref. 7.

Oblique Detonation Wave Relations

The laws of conservation of mass, momentum, and energy, applied to the plane oblique exothermic discontinuity considered, yield the relationships,

Continuity:

$$\rho_j u_j = \rho_i u_i \tag{1}$$

Momentum:

$$p_j + \rho_j u_j^2 = p_i + \rho_i u_i^2 \tag{2}$$

and

$$v_j = v_i \tag{3}$$

Energy:

$$\frac{u_j^2 + v_j^2}{2} + \frac{a_j^2}{\gamma_j - 1} + q_{ij} = \frac{u_i^2 + v_i^2}{2} + \frac{a_i^2}{\gamma_i - 1} \tag{4}$$

The premixed gaseous mixture of reactants and the gaseous reaction products are assumed to be perfect gases with equations of state of the form

$$p_k = \rho_k R_k T_k \tag{5}$$

Manipulation of the above equations yields the following relationships for the determination of the flow variables behind the detonation wave in functions of the pressure ratio $P_{ij} = p_i / p_j$.

Normal velocity ratio (or density ratio):

$$\frac{\rho_j}{\rho_i} = \frac{u_i}{u_j} = \frac{P_{ij} + b_j + \bar{Q}}{b_i P_{ij} + 1}; \quad \bar{Q}_{ij} = \frac{2\gamma_j}{(\gamma_j - 1)} Q_{ij}$$

$$Q_{ij} = \frac{q_{ij}}{c_{p_j} T_j}; \quad b_k = \frac{\gamma_k + 1}{\gamma_k - 1} \tag{6}$$

Temperature ratio:

$$\frac{T_i}{T_j} = \frac{R_j}{R_i} P_{ij} \frac{P_{ij} + b_j + \bar{Q}_{ij}}{b_i P_{ij} + 1} \tag{7}$$

Mach number ratio:

$$\frac{M_i}{M_j} = \left(\frac{\gamma_j}{\gamma_i} \frac{1}{P_{ij}} \left\{ 1 - \frac{P_{ij} - 1}{\gamma_j M_j^2} \left(2 - \frac{P_{ij} - 1}{\gamma_j M_{jn}^2} \right) \right\} \Big/ \left[1 - \frac{P_{ij} - 1}{\gamma_j M_{jn}^2} \right] \right)^{1/2} \tag{8}$$

where the normal component of the Mach number of the flow before the detonation wave M_{jn} is given by

$$M_{jn}^2 = \frac{(P_{ij} - 1)(b_i P_{ij} + 1)}{\gamma_j \left(\frac{2}{\gamma_i - 1} P_{ij} - \frac{2}{\gamma_j - 1} - \bar{Q}_{ij} \right)} \tag{9}$$

the normal Mach number ratio is

$$\frac{M_{in}}{M_{jn}} = \sqrt{\frac{\gamma_j}{\gamma_i} \frac{1}{P_{ij}} \frac{P_{ij} + b_j + \bar{Q}_{ij}}{b_i P_{ij} + 1}} \tag{10}$$

the total pressure ratio

$$\frac{p_i^0}{p_j^0} = P_{ij} \frac{\left[1 + \frac{\gamma_i - 1}{2} M_j^2 \left(\frac{M_i}{M_j} \right)^2 \right]^{\gamma_i / (\gamma_i - 1)}}{\left(1 + \frac{\gamma_j - 1}{2} M_j^2 \right)^{\gamma_j / (\gamma_j - 1)}} \tag{11}$$

and the total temperature ratio

$$\frac{T_i^0}{T_j^0} = \frac{\gamma_j (\gamma_i - 1)}{\gamma_i (\gamma_j - 1)} \frac{R_j}{R_i} \left[1 + \frac{Q_{ij}}{1 + M_j^2 (\gamma_j - 1)/2} \right] \tag{12}$$

For strong detonations considered here, the ratio $P_{ji} = p_j / p_i$ will vary in the interval

$$0 < P_{ji} < P_{jiCJ} \tag{13}$$

where $P_{jiCJ} = 1/P_{ijCJ}$ can be determined from Eqs. (9) and (10)

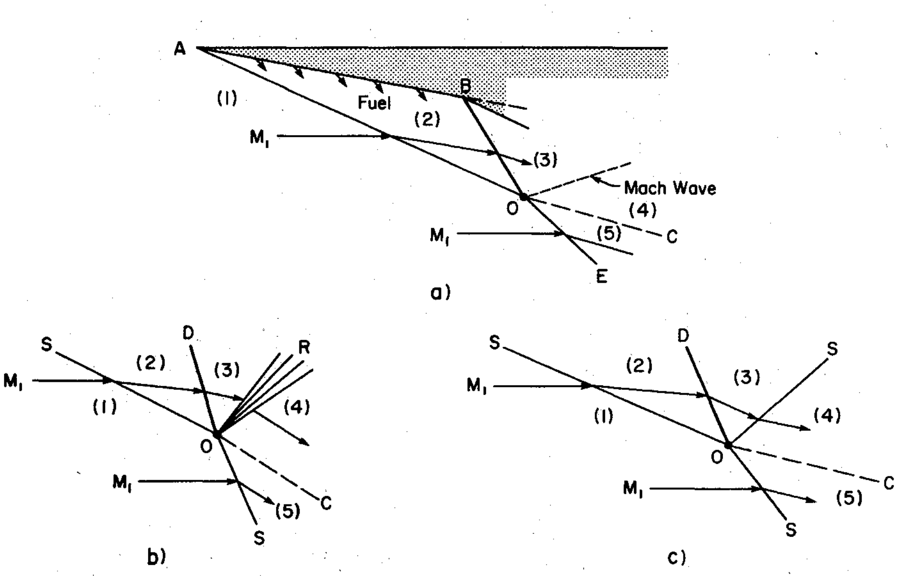


Fig. 1 Possible discontinuity configurations resulting from the interaction of a shock wave with a detonation wave (S = shock wave, D = detonation wave, R = rarefaction wave, C = contact discontinuity): a) configuration with a reflected Mach wave; b) configuration with a reflected rarefaction wave; c) configuration with a reflected shock wave.

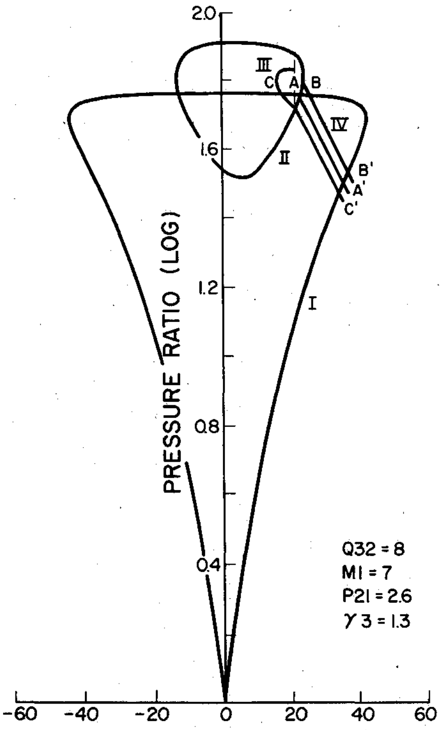


Fig. 2 Interaction of a detonation wave with a shock wave for different values of P_{32}.

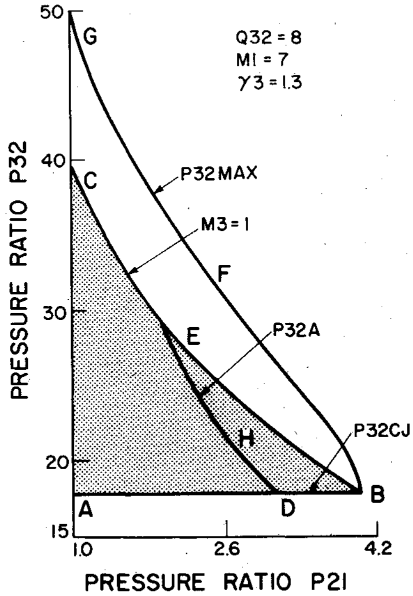


Fig. 3 Domain of existence (shaded area) of interaction configurations with a reflected rarefaction wave in the P_{21}, P_{32} plane.

with $M_{in} = 1$. We get

$$P_{ijCJ} = \left\{ \frac{\gamma_i + 1}{\gamma_j - 1}\left(1 + \frac{\gamma_j - 1}{2}\bar{Q}_{ij}\right) + \sqrt{\left[\frac{\gamma_i + 1}{\gamma_j - 1}\left(1 + \frac{\gamma_j - 1}{2}\bar{Q}_{ij}\right)\right]^2 - b_i(b_j + \bar{Q}_{ij})} \right\} \Big/ b_i \tag{14}$$

The flow deflection angle is given by

$$\delta_{ij} = \pm\tan^{-1}\left\{ \frac{P_{ij} - 1}{[\gamma_j M_j^2 - (P_{ij} - 1)]} \times \sqrt{\frac{\gamma_j M_j^2[(b_i - 1)P_{ij} - (b_j - 1) - \bar{Q}_{ij}]}{(P_{ij} - 1)(b_i P_{ij} + 1)} - 1} \right\} \tag{15}$$

where the upper sign corresponds to a clockwise deflection. When $\gamma_i = \gamma_j$ and $q_{ij} = 0$, the above equations reduce to the usual oblique shock wave relations.

Basic Equations and Solution of the Interaction Problem

The three possible interaction configurations are depicted in Fig. 1. In regions 1-5 divided by the waves the flow is uniform. In terms of the pressure ratio P_{ij} and the flow deflection angle δ_{ij} across the wave separating region i after the wave from region j before it, we can write the following conditions:

$$P_{51} = P_{43}P_{32}P_{21} \tag{16}$$

and

$$\delta_{51} = \delta_{21} + \delta_{32} + \delta_{43} \tag{17}$$

valid across the contact discontinuity OC. The deflection angles in Eq. (17) are given by Eq. (15) with the appropriate values of γ_j, Q_{ij}, M_j, and P_{ij} on the corresponding discontinuity (in the case of a shock discontinuity, the relevant value of γ is assumed conserved across the shock wave and $Q_{ij} = 0$). In the case of a reflected rarefaction wave, the corresponding deflection angle is given by

$$\delta_{43} = b_3\tan^{-1}\sqrt{(M_4^2 - 1)/b_3} - \tan^{-1}\sqrt{M_4^2 - 1} - b_3\tan^{-1}\sqrt{(M_3^2 - 1)/b_3} + \tan^{-1}\sqrt{M_3^2 - 1} \tag{18}$$

where

$$M_4 = \sqrt{\frac{2}{\gamma_3 - 1}\left[\frac{1 + M_3^2(\gamma_3 - 1)/2}{P_{43}^{(\gamma_3 - 1)/\gamma_3}} - 1\right]} \tag{19}$$

Elimination of P_{43} from Eqs. (16) and (17) yields the following equations for the single unknown P_{51}:

$$\delta_{21}(P_{21},\gamma_1,M_1) + \delta_{32}[P_{32},\gamma_2,\gamma_3,M_2(P_{21},\gamma_1,M_1),\bar{Q}_{32}] \pm \delta_{43}\left\{\frac{P_{51}}{P_{32}P_{21}},\gamma_3,M_3[P_{32},\gamma_2,\gamma_3,M_2(P_{21},\gamma_1,M_1),\bar{Q}_{32}]\right\} - \delta_{51}(P_{51},\gamma_1,M_1) = 0 \tag{20}$$

For given M_1, P_{21}, P_{32}, γ_1, γ_3, and Q_{32} (given fuel/air mixture), P_{51} is determined from Eq. (20). Equations (6-15), (18), and (19) will then yield the values of all the flow variables of interest in the considered interaction problem. In the case of vanishingly small reflected wave (triple-discontinuity configuration) $P_{51} = P_{32}P_{21}$; Eq. (20) is then solved for P_{32} for given values of P_{21}.

A graphical illustration of the solution is presented on Fig. 2 for the case when $M_1 = 7$, $P_{21} = 2.6$ ($M_2 = 5.92$), $\gamma_3 = 1.3$, and $Q_{32} = 8$. Only the right-hand halves of the incident shock polar I and detonation polar II are considered in the present investigation, as it is assumed that the shock and detonation waves deflect the flow in the same direction. The intersection point A of the detonation polar II and the shock polar I would then represent the resulting interaction pattern with a reflected Mach wave (triple-discontinuity configuration, case a in Fig. 1). Hence, for values of $P_{32} > P_{32_A}$, the value of P_{32} at point A, we would have reflected rarefaction wave configurations (case b in Fig. 1) determined by points B and B′ of intersection of the corresponding epicycloid III with the shock polar I; and for $P_{32} < P_{32_A}$, either the reflected shock configuration (case c in Fig. 1) determined by point C of intersection of the shock polar IV drawn from point P_{32} and the shock polar I, or the reflected rarefaction wave configuration determined by point C′ of intersection of the epicycloid III drawn from point P_{32} and the shock polar I. For $P_{32} = P_{32_A}$, a reflected rarefaction wave interaction configuration given by point A′ of intersection of the epicycloid drawn from point P_{32_A} and the shock polar I is also possible. Thus for a given combination of P_{21} and P_{32}, it is possible to have two sets of solutions corresponding respectively to points A, B, C and A′, B′, C′ in Fig. 2. It is obvious that not all the solutions are physically realizable. Whether a mathematical solution of Eq. (20) is physically realizable or not depends on the stability of the resulting triple discontinuity configuration to small perturbations.

Stability Criterion for Reflected Mach Wave Configurations

Let us superimpose a small pressure disturbance Δp on flow regions 3-5 (Fig. 1a) downstream of the discontinuities OB and OE. This small pressure disturbance will cause variations in the flow deflection angles across the reflected Mach wave and the transmitted shock wave OE, $\Delta\delta_R$ and $\Delta\delta_T$, respectively, and will not affect the flow in regions 1 and 2. If the rate of change of the flow deflection angle across the reflected Mach wave is less than the rate of change of the flow deflection angle across the transmitted shock wave, i.e., if

$$\left.\frac{d\delta}{dp}\right|_R \leq \left.\frac{d\delta}{dp}\right|_T \tag{21}$$

then the reflected Mach wave configuration is stable and, hence, physically realizable. Indeed, if $\Delta p > 0$, then from Eq. (21), $\Delta\delta_R < \Delta\delta_T$ and the streamlines crossing these waves will diverge and result in a pressure decrease which will restore the equilibrium state. If $\Delta p < 0$, then $\Delta\delta_R > \Delta\delta_T$, and the streamlines will converge and result in a pressure increase which will restore the equilibrium state. It is easy to see that if Eq. (21) is violated, the reflected Mach wave configuration becomes unstable and hence physically not realizable. Equation (21) can be written in a more convenient form

$$\left.\frac{d\delta}{d\ell n p}\right|_R \leq \left.\frac{d\delta}{d\ell n p}\right|_T \tag{22}$$

Substitution of the expressions for the derivatives in Eq. (22) yields

$$\left\{P_{51}\Big/\left[\frac{A(P_{51}-1)^2(P_{51}-P_{51_{max}})}{[\gamma_1 M_1^2-(P_{51}-1)]^2(AP_{51}+1)}-1\right]\right\}\left(\frac{\gamma_1 M_1^2}{[\gamma_1 M_1^2-(P_{51}-1)]^2}\left[-\frac{A(P_{51}-P_{51_{max}})}{(AP_{51}+1)}\right]^{1/2}\right.$$

$$\left.-\frac{A(AP_{51}+1)(2P_{51}-1-P_{51_{max}})-(P_{51}-P_{51_{max}})(2AP_{51}-A+1)}{2(AP_{51}+1)^2[\gamma_1 M_1^2-(P_{51}-1)]\left[-\dfrac{A(P_{51}-P_{51_{max}})}{(AP_{51}+1)}\right]^{1/2}}\right\} \leq \frac{(M_3^2-1)^{1/2}}{\gamma_3 M_3^2} \tag{23}$$

where

$$P_{51_{max}} = \frac{\gamma_1-1}{2(\gamma_1+1)}\left\{\frac{2\gamma_1 M_1^2}{\gamma_1-1}+\frac{2}{\gamma_1-1}\right.$$

$$\left.+\left[\left(\frac{2\gamma_1 M_1^2}{\gamma_1-1}+\frac{2}{\gamma_1-1}\right)^2-4A\left(\frac{2\gamma_1 M_1^2}{\gamma_1-1}-1\right)\right]^{1/2}\right\}$$

and

$$A = \frac{\gamma_5+1}{\gamma_5-1} = \frac{\gamma_1+1}{\gamma_1-1}$$

Equation (23) is the necessary and sufficient condition for the stability of the resulting limiting triple discontinuity configuration of the interaction problem considered.

Because the slope of the isentrope in the δ,p plane, $(d\delta/d\ell n p)_R$, is always negative, Eq. (23) is always satisfied for mathematical solutions with a reflected Mach wave which are on the weak branch of the transmitted shock polar, where $(d\delta/d\ell n p)_T \geq 0$ always. However, Eq. (23) can also be satisfied on a certain portion of the strong branch of the transmitted shock polar, where $(d\delta/d\ell n p)_T < 0$, resulting in physically realizable reflected Mach wave configurations. These triple-discontinuity configurations represent the limiting cases for physically possible interaction patterns with reflected shock or rarefaction waves.

Equation (23) is not satisfied at point A in Fig. 2. Hence, the reflected Mach wave interaction configuration represented by this point for the flow conditions considered, as well as points C and B corresponding to reflected shock and rarefaction wave configurations, respectively, are not physically realizable. The only stable and physically possible solutions are given in this case by points B′, A′, and C′.

Analysis of Wave Configurations

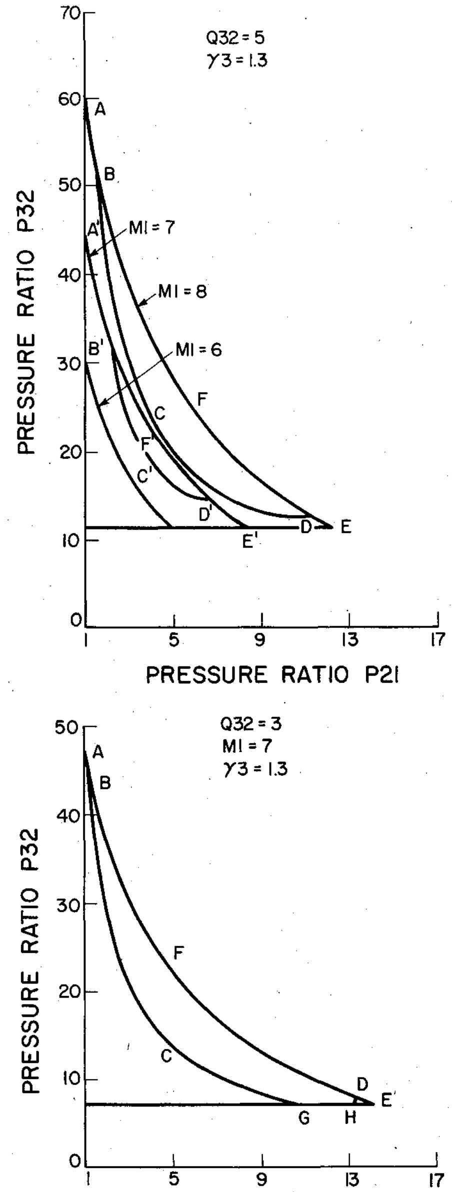
The intervals within which P_{21} and P_{32} can vary depend on the oncoming flow Mach number M_1 and the fuel/air mixture considered, i.e., on γ_2, γ_3, and Q_{32}. The procedure for determining these intervals, as well as the domains of existence of the resulting physically possible interaction configurations and the strengths of the resulting waves, is here given for the particular case when $M_1 = 7.0$, $\gamma_1 = \gamma_2 = 1.40$, $\gamma_3 = 1.30$, and $Q_{32} = 8.0$.

It is obvious that the lower limit of the interval of variation of P_{21}, $P_{21_\ell} = 1$. The upper limit of this interval P_{21_u} is determined by the condition that for given M_1, γ_2, γ_3, and Q_{32} there is a Mach number M_2 in region 2 for which the detonation wave is operating at the single Chapman-Jouguet condition (i.e., the detonation polar in the (p, δ) plane shrinks to a point). Letting $M_{n_3} = M_3 = 1$ in Eqs. (8-10) and eliminating M_{n_2} and P_{32} from these equations, we get the following equation to determine M_2:

$$\frac{1}{(\gamma_3-1)^2}(\gamma_2 M_2^2+1)^2 - \frac{\gamma_3+1}{\gamma_3-1}\left[\frac{2\gamma_2 M_2^2}{\gamma_2-1}(1+\gamma_2 Q_{32})-1\right]=0 \tag{24}$$

Elimination of M_{2_n} from Eqs. (8) and (9) will then yield an equation from which the value P_{21_u} is computed. For the numerical example considered $M_2 = 5.4$ and $P_{21_u} = 4.04$.

For strong detonation waves considered in the present paper the lower limit of the interval of variation of P_{32}, P_{32_ℓ} should be its Chapman-Jouguet value given by Eq. (14), $P_{32_\ell} = P_{32_{CJ}}$ (= 17.85 for the numerical example considered). The upper limit for the variation of P_{32}, P_{32_u} is given by the condition $M_3 = 1$. Eliminating again M_{2_n} from Eqs. (8) and (9) and letting $M_3 = 1$, we arrive at an equation from which the value $P_{32_u} = P_{32_s}$, i.e., the value of P_{32} giving sonic flow behind the detonation wave is obtained. It should be noted that this value of P_{32_s} is a function of P_{21}. Figure 3 depicts the domain of variation of P_{21} and P_{32} for the specific numerical



a)

b)

Fig. 4 Domain of existence of interaction configurations with a reflected rarefaction wave in the P_{21}, P_{32} plane for different values of the oncoming flow Mach number M_1 (areas under the corresponding curves).

example treated (shaded area). The horizontal line AB represents the lower, constant Chapman-Jouguet limit of P_{32}. The curve BEC is the locus of values of P_{32_s} and the curve BFG that of $P_{32_{max}}$ (resulting in normal detonation waves). The curve DHE is the locus of pairs of values of P_{21} and P_{32} of the strength of the interacting shock and detonation waves which result in a configuration with a reflected Mach wave.

Numerical calculations show that for all of these solutions on line DHE Eq. (23) is not satisfied. Therefore, they are not physically realizable. On the other hand, calculations show that, for every point in the domain ABEC (Fig. 3), there is a solution with a reflected rarefaction wave.

Similar calculations were performed for different values of M_1 and Q_{32}. The corresponding domains of existence of interaction configurations with a reflected rarefaction wave are presented in Fig. 4 (areas under the corresponding curves).‡ It should be noted that, as the values of the heat release

‡Similar domains of existence are reported in Ref. 8 for the range of values: $M_1 = 6$-8, $Q_{32} = 3$-8, and $\gamma_5 = 1.30$, as well as the strengths of the transmitted and rarefaction waves for various values of the strengths of the interacting waves.

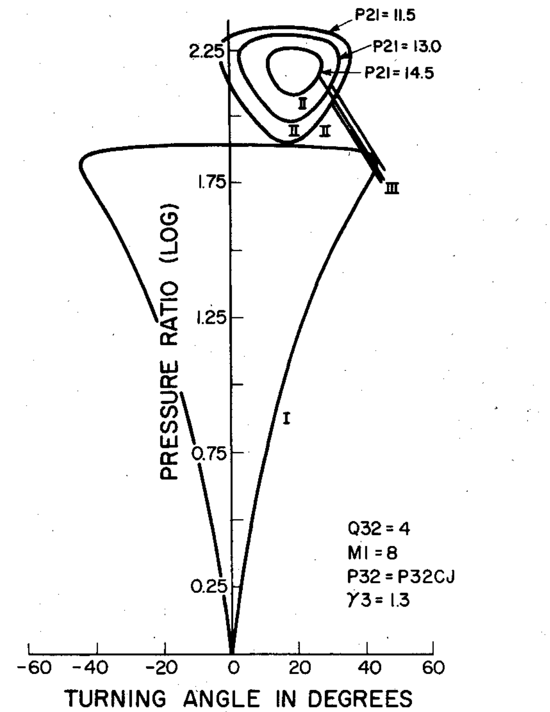


Fig. 5 Discontinuity of the interval of P_{21} because of absence of intersection between the epicycloid of the reflected rarefaction wave and the transmitted shock polar.

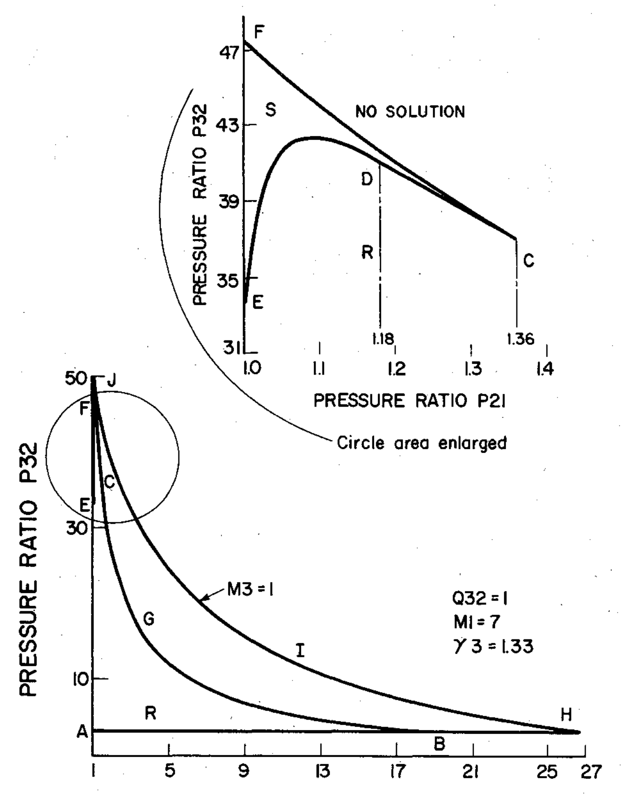


Fig. 6 Domains of existence of various interaction configurations (S—with a reflected shock wave, R—with a reflected rarefaction wave).

parameter are decreased, the upper limit of the interval of variation of P_{32} ceases to be P_{32_s} because the epicycloid III describing the rarefaction wave issuing from the point P_{32_s} on the detonation polar II does not intersect the shock polar I. Therefore, there is no solution of the interaction problem. In these cases the upper limit of the interval of variation of P_{32} is determined numerically as the point where the roots of Eq.

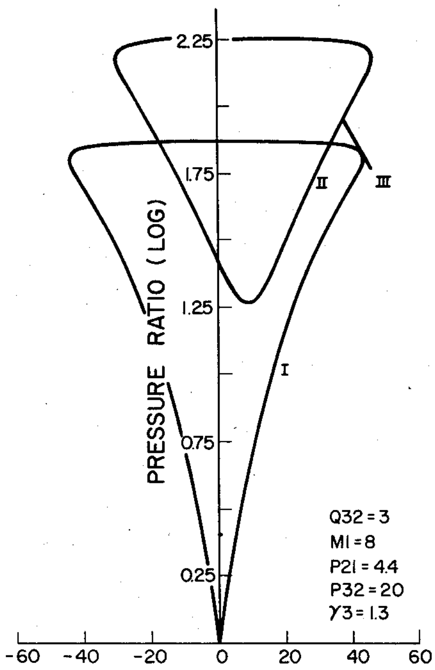


Fig. 7 Interaction between detonation wave and shock wave with transmitted shock wave on the strong branch.

(20), if they exist, coincide. The domains of existence of reflected rarefaction wave configuration are then represented by the areas under the lines ABCDE and A′B′C′D′E′ in Fig. 4a.

If we further decrease the value of Q_{32} (for example, $Q_{32}=3$ and $M_1=7$), the aforementioned domain of existence splits in two subdomains ABCGK and DHE (see Fg. 4b). There are no solutions for P_{21} in the interval GH and for $P_{32} \geq P_{32_{CJ}}$. The reason for this splitting is clear from the graphical solution presented in Fig. 5. This figure depicts the case when P_{32} is kept constant at $P_{32}=P_{32_{CJ}}$ and P_{21} is assigned the values 11.5, 13.0, and 14.5. The corresponding epicycloids for the reflected rarefaction waves issuing from points $P_{32}=P_{32_{CJ}}$ intersect the shock polar I for $P_{21}=11.5$ and 14.5 but not for the intermediate value $P_{21}=13.0$.

Stable, and hence physically possible, reflected Mach wave configurations have been found to exist for very low values of the heat release parameter Q_{32}. Figure 6 presents results of numerical calculations for the case $M_1=7$, $\gamma_1=\gamma_2=1.4$, $\gamma_3=1.33$, and $Q_{32}=1.0$. Curve EDC is the locus of stable [according to Eq. (23)], and hence physically realizable, reflected Mach wave configurations separating regions where stable interaction configurations with reflected shock (area EDCF) and rarefaction waves (area EDCGBA) occur. Points on the ED portion of this curve correspond to configurations with transmitted shock waves on the weak branch of the shock polar, whereas for points on the DC portion the transmitted shock wave is on the strong branch of the shock polar. There are no regular interaction solutions in the region above the curve BGCF. Curve HIJ is the locus of points for which the Mach number behind the detonation wave is exactly sonic, $M_3=1$.

It is of interest to consider the particular case when $P_{21}=1.0$ always (line AEF in Fig. 6). The problem then reduces to the refraction of a detonation wave at a combustible/noncombustible gas interface. The equilibrium of pressures behind the detonation wave and the shock wave (to which the detonation wave degenerates in the noncombustible gas mixture) is achieved through a shock wave if the strength of the detonation wave P_{32} lies in the interval EF, and through a rarefaction wave if P_{32} is in the interval AE. Point E corresponds to a double-discontinuity situation, i.e., only the detonation and shock waves are present, the equilibrium of pressures taking place across a Mach wave. Thus for the case considered (where $\gamma_1=\gamma_2=\gamma_5 \neq \gamma_3$) such a double-discontinuity configuration, with energy addition on one of the discontinuities, is possible. Rues[6] has shown that for $\gamma_1=\gamma_2=\gamma_3=\gamma_5$ such a configuration is impossible. Calculations performed for the particular case considered, assuming constant specific ratio everywhere, have also shown that such a double-discontinuity configuration does not exist. This example again shows the importance of the value of the specific heat ratio of the combustion products γ_3 for the interaction problem considered.

Discussion and Conclusions

The transmitted shock wave is usually on the weak solution branch of the shock polar. However, when P_{32} approaches P_{32_u}, the transmitted shock wave may be on the strong solution branch, as exemplified by Fig. 7. Both theoretical solutions are on the strong branch of the shock polar with $P_{51}=66.44$ and 65.46; the latter solution is physically possible according to solution and continuity arguments.

The upper limit of the interval of variation of P_{21}, P_{21_u} is fixed by the Chapman-Jouguet condition of the detonation wave. The higher the oncoming flow Mach number and the lower the value of the heat release parameter of the detonation wave, the larger P_{21_u} is. When P_{21_u} is large, for some values of the incident shock wave strength P_{21} there is no solution to the regular interaction problem considered and the interval of variation of P_{21} becomes discontinuous.

In the problem of the interaction of a shock wave with a detonation wave which deflects the flow in the same direction, the determination of the nature of the reflected wave is of primary interest. It has been found that for most combustible mixtures of gases ($3 \leq Q_{32} \leq 8$, $1.30 \leq \gamma_3 \leq 1.33$) the reflected wave is always a rarefaction wave. Using Eq. (23), it has been shown that triple-discontinuity configurations (reflected Mach wave) and hence configurations with a reflected shock wave and rarefaction wave are physically possible for combustible gases with low heat release and are very sensitive to the values of the specific heat ratio of the combustion products behind the detonation wave. Considering the particular case of refraction of a detonation wave at a combustible/noncombustible gas interface, it has been found that for low values of Q_{32} ($Q_{32}=1.0$) and $\gamma_1=\gamma_2=\gamma_5 \neq \gamma_3$ double-discontinuity configurations, where only the detonation and shock waves are present (the equilibrium of pressures behind the detonation and shock waves takes place across a Mach wave), can exist.

In general, the magnitudes of the heat release parameter Q_{32} and specific heat ratio γ_3 of the combustion products behind the detonation wave affect significantly the interaction pattern of shock and detonation waves. Hence, for a given particular flow configuration of the interacting shock and detonation waves and combustible mixture of gases, we recommend that a detailed thermochemical analysis be made in order to determine the actual values of the heat release parameter Q_{32} and the specific heat ratio of the combustion product γ_3. The nature of the resulting interaction pattern could then be accurately established by using these actual values of Q_{32} and γ_3.

The interaction problem considered in the present study was also investigated by Rues[6] for the particular case when the resulting interaction pattern involves only a reflected Mach wave; no allowance was made for the change of specific heat ratios across the detonation wave. For this particular situation, the results obtained in the present work coincide with those of Ref. 6.

However, according to Eq. (23), all triple-discontinuity configurations studied by Rues are not stable and hence physically not realizable. When $Q_{32}=0$ and $\gamma_2=\gamma_3=1.4$, the present results coincide with those of Ref. 9, where the similar interaction problem of two shock waves is considered.

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